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The Challenging Cosmic Singularity*

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In this talk we discuss the cosmic singularity. We motivate the need to correct general relativity in the study of singularities, and mention the kind of corrections provided by string theory. We review how string theory resolves time-like singularities with two examples. Then, a simple toy model with lightlike singularities is presented, and studied in classical string theory. It turns out that classical string theory cannot resolve these singularities, and therefore better understanding of the full quantum theory is needed. The implications of this result for the Ekpyrotic/Cyclic Model are discussed. We end by mentioning the known suggestions for explaining the cosmological singularity.

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1. Introduction

For over thirty years the singularity theorems of Hawking and Penrose have presented a challenge to fundamental physics [1,2]. Among the most dramatic consequences of these theorems are the implications for the standard big bang cosmology: If we extrapolate cosmological evolution back in time, we are driven to an initial singularity where all known physical laws break down. The existence of the big bang singularity raises some of the most challenging questions in physics: Is there a beginning of time? What happens before the big bang? Are such questions even meaningful and susceptible of scientific enquiry?

A proper understanding of the singularities requires going beyond general relativity and introducing new laws of physics. One promising avenue to doing so is string theory. String theory is a consistent and finite theory of quantum gravity which reduces to Einstein's general relativity at low energies and long distances. It is therefore natural to ask whether, and if so, how, a singularity in general relativity is resolved in string theory. By "resolved" we mean that the theory yields well-defined finite answers to physical questions. One hopes that string theory might lead to a detailed theory of the big bang. This in turn might lead to observable signatures for quantum gravitational physics and experimental tests of string theory.

2. Resolution of timelike singularities in string theory

Over the last two decades advances in string theory have provided us with various mechanisms for understanding the resolution of time-independent singularities. These provide essential paradigms for singularity-resolution in string theory and we will therefore briefly review two central examples of such singularity resolution before discussing the big-bang type singularities. In order to appreciate these resolutions we must first recall the relation of string theory to general relativity.

At low energies and weak string coupling, the classical and quantum stringy corrections appear as a generally covariant extension of the Einstein-Hilbert action with higher curvature interactions. More precisely, the modifications can be understood as a double expansion in α'/l_c^2 and g_s. α' is the inverse of the string tension, and l_c is a typical curvature radius. The string coupling constant plays the role of $\hbar$ and also serves as the loop counting parameter in string perturbation theory. For example, with four non-compact

dimensions, the low energy effective action of string theory in the absence of a cosmological constant has the form

$$\begin{aligned} S_{eff} = & \frac{1}{g_s^2\alpha'} \int d^4x\sqrt{-g}\left(R + a_2\alpha' R^2 + \cdots\right) \\ & + \frac{1}{\alpha'} \int d^4x\sqrt{-g}\left(b_1 R + b_2\alpha' R^2 + \cdots\right) \\ & + \cdots\cdots \end{aligned} \tag{2.1}$$

with the four-dimensional Planck constant identified as

$$M_{pl}^2 = \frac{1}{g_s^2\alpha'}\left(1 + b_1 g_s^2 + \cdots\right) \ . \tag{2.2}$$

The first line in (2.1) corresponds to the classical contribution. The second line is the first order quantum correction. The expansion in $\alpha' R$ can be considered as due to the extended nature of the string; the $g_s = \hbar$ corrections arise from quantum effects. In particular, the higher order terms in the first line in (2.1) correspond to the classical string modifications of the Einstein gravity. There is no arbitrary UV cutoff in (2.1). Rather, string theory provides a natural "UV cutoff" in the string scale; this is one of the keys to the UV finiteness properties of the theory, resolving the long-standing puzzle of nonrenormalizability of Einstein gravity.

At very low energies or low curvature, and weak string coupling, the first term in (2.1) dominates and one recovers the Einstein action. However, at high energy or around regions of spacetime with large curvature, in which

$$E^2\alpha' \sim O(1), \qquad \text{or} \qquad \frac{E^2}{M_{pl}^2} \sim O(1) \tag{2.3}$$

all higher order terms in (2.1) become important. The low energy expansion is no longer valid and an exact stringy treatment is needed. In the two examples below the expansion (2.1) breaks down. Nevertheless, the string theory is susceptible to an exact treatment, and a satisfying consequence of this treatment is an understanding of singularity resolution.

In both examples below the key to singularity resolution is that, due to the presence of the spacetime singularity, new degrees of freedom in string theory become important. These new degrees of freedom can be classified as *perturbative* or *non-perturbative* degrees of freedom, according to how the masses depend on the string coupling constant g_s. The masses of perturbative string states have a smooth limit when we take $g_s \to 0$. These

states can be understood as oscillation and winding modes of fundamental strings. The non-perturbative states have masses varying as an inverse power of the string coupling. Therefore, in smooth backgrounds and at weak coupling $g_s \ll 1$, the non-perturbative degrees of freedom are rather heavy, they do not contribute in perturbation theory, and their effects are in general small and relatively unimportant. By contrast, around spacetime singularities where the expansion (2.1) breaks down such non-perturbative states actually become important. Examples of such non-perturbative states include 5-branes, D-branes and black holes.

Let us now turn to our two central examples. In the first example the singularity is resolved at the classical level, while in the second example a full non-perturbative treatment is required.

Example I: Orbifold singularity

Consider the two dimensional space obtained by identifying

$$(x_1, x_2) \qquad \to \qquad (-x_1, -x_2) \tag{2.4}$$

in Euclidean $\mathbb{R}^2$. The resulting space is a two dimensional cone with a deficit angle π. Classical general relativity is singular in this background because of the delta function curvature at the tip of the cone. Quantum field theory is also singular in this background. Moreover, because of the singular curvature at the orbifold fixed point the expansion (2.1) breaks down. Nevertheless, string theory is solvable in this background, and it turns out that the extended nature of the strings leads to new degrees of freedom, known as "twisted states," which are localized near the singularity. These degrees of freedom make the classical as well as the quantum physics completely smooth [3,4]. Roughly speaking, since the string is extended, the singularity at the tip of the cone becomes fuzzy and regular. This is an example of a singularity of classical general relativity and QFT which is resolved by classical string effects.

Example II: Conifold singularity

Let us now consider another example of an orbifold singularity, the so-called A_1 singularity, obtained by taking a Z_2 quotient of Euclidean $\mathbb{R}^4$

$$(x_1, x_2, x_3, x_4) \qquad \to \qquad (-x_1, -x_2, -x_3, -x_4) \ . \tag{2.5}$$

As a Riemannian manifold, the space is singular at the origin of $\mathbb{R}^4$, the fixed point of the identification. In classical general relativity the singularity at $x^\mu = 0$ is a bolt singularity

and can be "blown up" to a spacetime with a noncontractible two-sphere, S^2, of radius R. The orbifold geometry is then recovered as $R \to 0$. General relativity and QFT are well-defined for $R > 0$, but become singular as $R \to 0$. It turns out that, again, the first quantized perturbative string theory is smooth in this background, even when $R = 0$, and again the underlying reason is the extended nature of the string. In this case, in addition to the twisted states there is another crucial ingredient in the resolution. In string theory a "geometry" or "background" is not simply specified by a Riemannian metric. There are other quantities which must be specified, the most important in the present example being the value of an harmonic 2-form commongly denoted $B_{\mu\nu}$. The integral

$$B = \int_{S^2} \frac{1}{2} B_{\mu\nu} dx^{\mu} \wedge dx^{\nu} \tag{2.6}$$

is independent of R, and is a crucial part of the data specifying a stringy background. When $R = 0$ and $B \neq 0$ the classical string theory is nonsingular. Thus far, the story is identical to Example I. However, when $R, B \to 0$ there are further divergences and *classical* string theory turns out to be singular. Nevertheless, in this case, it turns out that *quantum* string theory is smooth. The essential point is that one must take into account new *non-perturbative* degrees of freedom which become massless as $R, B \to 0$ [5,6].

To summarize, string theory introduces new degrees of freedom. By including them the physics at timelike singularities becomes regular. These singularities arise in general relativity simply because the relevant degrees of freedom have not been taken into account.

3. Cosmological singularities in string theory

Let us now turn to spacelike and lightlike singularities in string theory. Such cosmological singularities are much harder to understand, since the singularity appears in the past or in the future. Therefore, study of these singularities requires understanding time-dependent backgrounds - an aspect of string theory that has been somewhat neglected until recently.

To understand the cosmological singularity in string theory, we again have to face the question: Is the cosmological singularity resolved by classical string theory or by quantum string theory? In this talk we only address the first part of the question, i.e. we study the issue in string perturbation theory.

As a toy model, we consider a spacetime which is obtained from the $(2+1)$ flat Minkowski spacetime $\mathbb{R}^{1,2}$

$$ds^2 = -2dx^+dx^- + dx^2 \tag{3.1}$$

by discrete identifications [7]

$$\begin{aligned} &x^+ \sim x^+ \\ &x \sim x + 2\pi n x^+ \\ &x^- \sim x^- + 2\pi n x + \frac{1}{2}(2\pi n)^2 x^+ \end{aligned} \tag{3.2}$$

with $n = \pm 1, \pm 2, \cdots$. These identifications are generated by a Lorentz transformation which is a linear combination of a boost and a rotation.

To understand better the geometry of the resulting spacetime, it is convenient to use the coordinates

$$\begin{aligned} x^+ &= y^+ \\ x &= y^+ y \\ x^- &= y^- + \frac{1}{2}y^+ y^2 \end{aligned} \tag{3.3}$$

in terms of which the metric takes on a gravitational plane wave form:

$$ds^2 = -2dy^+dy^- + (y^+)^2 dy^2 \tag{3.4}$$

subject to the simple identiication $y \sim y + 2\pi n$. In terms of these coordinates it is manifest that the quotient spacetime is made out of two cones. One of them has $y^+ = x^+$ positive and the other has $y^+ = x^+$ negative. The radial direction of the cone $x^+ = y^+$ is a null coordinate in the full spacetime. The two cones touch at $y^+ = x^+ = 0$ where the space is singular. Note that the singularity is neither spacelike nor timelike; it is lightlike. [1]

The above spacetime has a few attractive features:

1. In (3.4) a circle of an infinite size at $y^+ = -\infty$ shrinks to zero size at $y^+ = 0$, and then expands to an infinite size again at $y^+ = \infty$. This provides an interesting toy model for understanding the big crunch/big bang singularity in cosmology.
2. Time-dependent orbifolds similar to (3.2) have been used recently in some proposed cosmological scenarios [8,9,10].

[1] We are skating over several important technicalities in this description. For example, the coordinate transformation (3.3) is ill defined at $x^+ = 0$. Indeed, it turns out that in X-coordinates the quotient is non-Hausdorff at $x^+ = 0$.

3. As a quotient of flat Minkowski spacetime, the orbifold (3.4) is amenable to exact perturbative string analysis since we can follow the standard procedure of [3,4,11]. Furthermore, this orbifold has some nice properties which make the stringy analysis particularly clean: it is supersymmetric, there are no closed time-like curves and there is no particle production.
4. The same type of singularity also appears in certain black holes (a closely related problem) as well as in more complicated cosmological backgrounds.

Since timelike orbifold singularities are neatly resolved by classical string theory, as in Example I above, it is natural to ask if spacelike or lightlike orbifold singularities are likewise resolved by classical string theory. We should note at the outset that there are important qualitative differences between the singularity of Example I and the cosmological singularity at $x^+ = 0$ of (3.2). For example, in the latter case, the point $X^\mu = 0$, is fixed by an *infinite* number of group elements. Thus, analogies between timelike and lightlike (or spacelike) orbifolds should be drawn with care.

In order to explore whether it is possible to pass through the singularity at $x^+ = 0$, i.e. whether one can evolve smoothly from a big crunch to a big bang we will first assume that one can indeed pass through the singularity. That is, we assume that classical string theory does indeed somehow resolve the lightlike singularity. In this case we should be able to compute S-matrix elements for particles going from the past cone $x^+ < 0$ to the future cone $x^+ > 0$. More precisely, we will start in the far past $x^+ \to -\infty$ with some particles (small fluctuations) and compute the amplitude to find other particles (other small fluctuations) in the far future $x^+ \to \infty$ using the standard rules for orbifolding string backgrounds. If the singularity is resolved, then one should be able to find a sensible S-matrix. On the other hand, a singular S-matrix would imply the breakdown of the formalism. In [12,13] we have looked at certain S-matrix elements in detail at leading order of string perturbation theory. We found that

- For generic kinematics the amplitudes in classical string theory are finite.
- However, for special kinematics (near forward scattering) the string amplitudes diverge.

We should note that, in general relativity on the orbifold (3.2) the amplitudes are divergent for generic kinematics. The finiteness of tree level string amplitudes for generic kinematics may be attributed to the softness of strings at high energies. However, the divergence in the forward scattering signals the breakdown of perturbation theory. In fact, the situation is potentially worse - it is entirely possible that at higher orders of string

perturbation theory the amplitudes are divergent for generic kinematics. Whether or not this is so is, at present, unknown.

The physical reason for the divergence is easy to understand [13-16]. Since the background depends on time, energy is not conserved. In particular the energy of an incoming particle is blue-shifted to infinity by the contraction at the singularity. The infinite energy of the incoming particles generates an infinitely large gravitational field and distorts the geometry. The perturbation series breaks down as a result of this large backreaction.

The most likely conclusion we should draw from these computations is that classical string theory need not resolve the singularities of time-dependent orbifolds. These classical solutions are unstable and we need to understand the full quantum theory to explain the physics at the singularity.

We say "most likely" because a number of open issues have yet to be settled. For examples, the possible effects of brehmstrahlung of light twisted sector states have not yet been carefully studied. Moreover, the orbifold admits a deformation, known as the "nullbrane," analogous to the classical deformation to $R > 0$ mentioned in Example II above [17,18,13,14,19]. String scattering in this background is generically nonsingular. The limit $R \to 0$ has not yet been carefully studied. For an entrée into the most recent literature, with references to many other papers related to our work see [20-22].

Let us now put the above computation in the perspective of enquiries into the nature of the big bang. The idea of going from a big crunch to a big bang through a non-singular bounce has a long history. In the thirties, Einstein considered the idea that a non-singular bounce might be achieved through irregularities. Richard Tolman considered in detail a cyclic universe using a similar mechanism. The singularity theorems of Hawking and Penrose [1,2] ruled out this possibility in general relativity. The recent suggestions [9] that the universe passes through the singularity are motivated by the orbifold construction of string theory. We now see that classical string theory is also singular in such orbifolds and cannot be trusted. Therefore, the rationale for the proposal of [9] is absent. This does not necessarily mean that the proposal is wrong; only that it is unmotivated.[2]

There are at least three possibilities for the interpretation of the singularity:

[2] The nullbrane at small radius has a nearly big crunch followed by a nearly big bang. It might be interesting to explore whether such a geometry is a reasonable substitute for the cyclic universe.

1. The singularity is a beginning or end of time. In this case we need to understand the appropriate initial conditions at the singularity. For some discussion of these issues see [23,24].
2. Time has no beginning or end. Then one needs to understand how to pass through the singularity (for recent discussions see e.g. [9,25]).
3. The most likely possibility, it seems to us,is that in string theory time is a derived concept.

In conclusion, let us elaborate on the third possibility above. In toroidal compactifications of string theory there is a minimal distance, thanks to T-duality: shrinking radii past the string scale does not produce a theory at shorter distances. In more elaborate compactifications (such as Calabi-Yau compactifications) it turns out that there can be smooth topology-changing processes, and "quantum geometry" can lead to many counter-intuitive types of behavior. These examples show that, in string theory, standard notions of topology and geometry are not fundamental but are rather emergent concepts in certain physical regimes (e.g. in regions of large complex and Kahler structure parameters, in the Calabi-Yau context). In another line of development, Matrix theory and the related advent of noncommutative field-theoretic limits of string theory further indicate that the notion of distance and space ceases to make sense in certain otherwise sensible regimes of the theory. Given the principle of relativity it seems quite likely, perhaps even inevitable, that similar statements hold for time as well as for space. That is, recent discoveries might be viewed as hints that evolution in time might be only an approximate, phenomenological, or emergent concept, which is only applicable in some but not all physical regimes. But what could we mean by "physics" without time? The notion of evolution in time lies at the very heart of classical mechanics, of classical (string) field theory and of the principles of quantum mechanics. Finding a satisfactory formulation of physics in which time evolution is truly an emergent concept is a worthy challenge to those who would challenge the standard paradigms of fundamental physics.

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